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Physical Constants as Cosmological Constraints

Saul-Paul Sirag

Washington Research Center, 3101 Washington Street, San Francisco, California 94115

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A solution to the primary "missing mass" problem is found in the context of accounting for the coincidence of large dimensionless numbers first noticed by Weyl, Eddington, and Dirac. This solution entails (1) a $\log_2$ relation between the electromagnetic and gravitational coupling constants; (2) setting the maximum radius of curvature at the gravitational radius, $2GM/c^2$; (3) a changing gravitational parameter G, which varies as an inverse function of the universal radius of curvature. These features motivate the development of a neo-Friedmann formalism, which employs a function, $\varepsilon(\chi)$, governing the change from Euclidian to non-Euclidian volumes. Observational consequences include (1) a universal density of 7.6×10^{-31}g cm^{-3}, (2) a Hubble parameter of 15 km s^{-1} Mpc^{-1}, (3) an age of the universe of 32×10^9 yr, (4) a gravitational parameter diminishing at a current rate of 2.2×10^{-12} yr^{-1}, and (5) a deceleration parameter of 1.93. Moreover, it is shown that for a Friedmann-type ($\Lambda = 0$) cosmology (whether open or closed) any deceleration parameter will be represented by a straight line in the (log–log) red shift: luminosity-distance space of the Hubble diagram. The major claim of this paper is that we have devised a model in which the large-scale structure of the universe is completely determined by the values of the fundamental physical constants: c, $\hbar$, e, and m_e setting the scale, and G selecting the epoch.

1. INTRODUCTION

The chief aim of observational cosmology today is to determine whether the universe is open or closed. A closed universe is defined as being topologically compact; and within the context of Friedmann cosmologies (which set the cosmological constant at zero) a closed universe has a maximum radius of curvature to which the universe expands from the "big bang" singularity, and from which it contracts to another singularity. If we assume a Friedmann cosmology, two measures are sufficient to decide

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between closure and nonclosure: ρ_0, the average density of matter in the universe today, and H_0, the Hubble parameter today. Recent measurements of these two quantities have led to the conclusion that the universe is open (i.e., that it will expand forever). Those who are partisans of a closed universe call this situation the "missing mass" problem (Misner, Thorne, and Wheeler, 1973).

Actually, there are other missing mass problems, such as the discrepancy between the quantity of visible galactic matter and the "virial mass" of galactic clusters (Field, 1976). However, this virial mass (even though it is greater than the visible mass) is still insufficient to close a Friedmann universe, given current measurements of H_0, so that the primary missing mass problem remains.

Hopes for revising ρ_0 upward have recently fluctuated: intergalactic gasses have been ruled out as a candidate for missing mass (Giacconi, 1980); and massive neutrinos have been raised to such candidacy (Reines, Sobel, Paseirb, 1980).

For the primary missing mass problem, however, the most relevant consideration is the Mathews and Viola (1979) determination of ρ_0 by the measurement of deuterium and lithium abundances (which is independent of H_0). These authors conclude that their value of $4.0 \times 10^{-31} < \rho_0 < 1.4 \times 10^{-30}$ g cm^{-3} (with best guess, 7.1×10^{-31} g cm^{-3}), even when combined with an H_0 as low as 40 km s^{-1} Mpc^{-1}, leaves the universe open.

While astronomers have been largely unprejudiced about the topology of the universe, theoretical physicists, beginning with Einstein (for reasons of elegance and utility), have tended to believe in a closed universe. Since there is observational conflict with a closed Friedmann cosmology, a determined partisan of universe closure might be expected to propose an alternative to Friedmann's equations. A major perplexity in pursuing this line of argument, however, has been the lack of natural constraints on possible cosmological models. The strategy of this paper will be to show that the physical constants as constituents of large dimensionless numbers provide natural constraints which require a closed universe. I will further show that, if in this context one postulates that the universe has a maximum radius of curvature equivalent to its gravitational radius, a present-day universal density of 7.6×10^{-31} is calculated, which accords with the observations of Mathews and Viola (1979), and thus eliminates the primary missing mass problem. (Since there is not enough visible matter to provide this density, a secondary missing mass problem remains, of course. And it is this problem that massive neutrinos would be most suited to solve.)

With the motivation provided by this matching calculation, I will then justify the postulated maximum radius formula by developing a neo-Friedmann formalism, from which I will calculate a present-day Hubble parameter of 15 km s^{-1} Mpc^{-1}. Since this and other observable consequences of the neo-Friedmann cosmology are in disagreement with present-day measurements, a decisive test of this model can be expected when the NASA Space Telescope (Tammann, 1979) refines distance parameters beginning about 1985.

2. LARGE DIMENSIONLESS NUMBERS

Weyl (1949), Eddington (1935), Dirac (1937), and others have remarked on the coincidence of large dimensionless numbers in physics, which can be succinctly formulated as

$$\frac{\alpha}{\gamma} \simeq \frac{R_h}{R_e} \simeq N^{1/2} \simeq 10^{40} \qquad (1)$$

where α is the electromagnetic coupling constant, $e^2/\hbar c = 1/137.03604$; and γ is the analogous gravitational coupling constant $Gm_p m_e/\hbar c = 1/3.1099 \times 10^{41}$ (where e is the electron charge, $\hbar$ is Planck's constant divided by 2π, c is the speed of light, G is the gravitational constant, m_p is the proton mass, and m_e is the electron mass). R_h is the Hubble radius of the universe, equivalent to c/H_0 (where H_0 is the present-day Hubble parameter); and R_e is the classical electron radius, equivalent to $e^2/m_e c^2$. N is the number of nucleons in the universe, estimated in various ways.

Dirac's (1937) "large-number hypothesis" proposes that these large-dimensionless number coincidences are physically meaningful. Since his formulation of these coincidences employs a time ratio, $1/H_0 \div R_e/c$ (which is equivalent to R_h/R_e), Dirac proposes that G diminishes as an inverse function of time and that matter is continuously created. This preserves the algebraic relations in equation (1), but it does not preserve the quantity, 10^{40}. Harrison (1972) has suggested a way to preserve both the algebraic relations and the quantity, 10^{40}, in equation (1). He proposes that the Hubble radius R_h be replaced by $R_{\max}$, the maximum radius of curvature in a closed Friedmann universe. Harrison argues that since the radius of curvature today, R_0 (which might be within an order of magnitude of R_h), could easily be around $\frac{1}{2}R_{\max}$, we would surely be struck by the coincidence of equation (1), whose ratios would thus be preserved at $\sim 10^{40}$.

Assuming that we accept the physical meaningfulness of equation (1), we still must ask if there is any way to decide between Dirac's continuous-creation proposal and Harrison's closed-universe proposal. We can easily decide in favor of Harrison if we can find a good reason to keep the ratios in equation (1) at $\sim 10^{40}$. I propose that there is another large-number coincidence which fulfills this requirement. Furthermore, I propose that this additional large-number coincidence provides a natural way to make the coincidences of ratios exact, and in so doing yields information sufficient for the calculation of the mass, present-day radius, and density of the universe—a density quite consistent with current universal density measurements.

The new large-number coincidence I wish to propose (since $2^{137} \simeq 10^{41}$) is

$$2^{1/\alpha} \simeq \frac{1}{\gamma} \simeq 10^{41} \tag{2}$$

Salam (1970) has proposed a logarithmic relation between α and γ. However, the $\log_2$ relation in equation (2) suggests a combinatorial Clifford algebra theory. Such a theory might be similar to that of Bastin (1971), but it should be noted that his $2^{127} \simeq 10^{38}$ relation employs the two-proton form of γ, and that he derives 137 from 127 by summing the subset series, 3, 7, 127, which is thus quite different from equation (2). My reason for saying that equation (2) suggests a Clifford algebra is quite straightforward, since, by definition (Lang, 1970), a Clifford algebra of dimension 2^n is generated on a vector space of dimension n. Of course, such a discrete approach would imply that the Clifford algebra formalism employs a discrete α, i.e., $1/\alpha_d = 137$ exactly. It is therefore fortunate that Burger (1978) has noted the Pythagorean relation:

$$(137.0360157)^2 = 137^2 + \pi^2 \tag{3}$$

so that a discrete algebraic formalism can be supposed to generate a continuous geometric formalism.

For our present purpose, we shall deal with the geometric side of the formalism and shall thus employ the traditional, nondiscrete α.

At this point, it is sufficient to realize that if (2) is physically meaningful, Dirac's proposal becomes untenable, and we therefore adopt Harrison's closed-universe proposal. Thus we can write

$$2^{1/\alpha} \simeq \frac{1}{\gamma} \simeq \frac{R_{\max}}{R_e} \simeq 10^{41} \tag{4}$$

Harrison assumed $R_{\max}$ to be $(4/3\pi)M^*$ (where $M^* \equiv GM/c^2$), as defined in the standard Friedmann formalism. However, if we define $R_{\max} \equiv R^*$ as the gravitational radius of the universe, $2M^*$ (which we will justify in the neo-Friedmann formalism), we can write

$$2^{1/\alpha} \simeq \frac{1}{\gamma} \simeq \frac{2Gm_p N}{c^2} \bigg/ \frac{e^2}{m_e c^2} \simeq 10^{41} \tag{5}$$

which is algebraically equivalent to

$$2^{1/\alpha} \simeq \frac{1}{\gamma} \simeq \frac{2\gamma}{\alpha} N \simeq 10^{41} \tag{6}$$

Thus we have incorporated all of the large-number coincidences of equations (1) and (2) into a simple, highly restrictive formulation, equation (6), from which we can easily derive

$$\frac{R^*}{R_e} \simeq \frac{1}{\gamma} \tag{6a}$$

$$N \simeq \alpha(2^{1/\alpha})^2 \tag{6b}$$

$$\alpha \simeq N\gamma^2 \tag{6c}$$

Since we have postulated that the large-number coincidences are physically meaningful, we are claiming, in effect, that any lack of exactness in equations (6)–(6c) must be due to physical factors—preferably a single physical parameter.

Accordingly, we note that because

$$2^{1/\alpha} \equiv 2^{\hbar c/e^2} = 1.7863 \times 10^{41} \tag{7}$$

and

$$\frac{1}{\gamma} \equiv \frac{\hbar c}{Gm_p m_e} = 3.1099 \times 10^{41} \tag{8}$$

then $2^{1/\alpha}$ equals $1/\gamma$ exactly if

$$\frac{1}{\gamma} = \frac{\hbar c}{(1.7410)Gm_p m_e} = 1.7863 \times 10^{41} \tag{9}$$

Thus the exact form of (6) would be

$$2^{1/\alpha} = \frac{1}{(1.7410)\gamma} = \frac{2\gamma}{\alpha} N = 1.7863 \times 10^{41} \tag{10}$$

Since we are looking for a single parameter to make this equation hold exactly, it seems obvious that the single parameter should be a changing G in the γ term. Indeed, the factors modifying the two γ terms in equation (10) suggest a very simple form for this hypothesized changing G. The 2γ term comes from the use of $R^* = 2GM^*$, which we have defined as $R_{\max}$ and is thus the radius of the universe at a particular time in the future, whereas the $(1.7410)\gamma$ term appears to reflect the condition of G today, which would be in process of becoming the G of the term 2γ.

Let us call G^* the G of 2γ, and G_0 the G of $(1.7410)\gamma$. Then, let us assume that G changes in the simplest way consistent with the requirement of making (10) exact. Thus we will propose that G is an inverse function of R, the radius of curvature of the universe. Accordingly,

$$\text{at } R_{\min} \equiv R', \qquad 2^{1/\alpha} = \frac{\hbar c}{G' m_p m_e} \tag{11}$$

$$\text{today at } R_0, \qquad 2^{1/\alpha} = \frac{\hbar c}{G_0 m_p m_e} \tag{12}$$

$$\text{at } R_{\max} \equiv R^*, \qquad 2^{1/\alpha} = \frac{\hbar c}{G^* m_p m_e} \tag{13}$$

where G_0 is the present-day gravitational "constant," 6.6720×10^{-8} cm^3 g^{-1} s^{-2}. Note that this formulation calls for a much smaller overall change in G than that proposed by Dirac, since we are requiring a change of only a factor of 2 over the entire duration of the evolution of the universe. That is,

$$0 < \frac{R}{R^*} < 1 \tag{14}$$

$$1 < \frac{G'}{G} < \frac{1}{2} \tag{15}$$

so that

$$\frac{R}{R^*} = \frac{G'}{G} - 1 \tag{16}$$

Thus equations (11)–(16) imply

$$G' = (1.7410)G_0 = 2G^* = 1.616 \times 10^{-7} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2} \tag{17}$$

so that

$$\frac{R_0}{R^*} = 0.7410 \tag{18}$$

It is useful, at this point, to define a set of γ terms in accord with equations (10)–(13):

$$\gamma' \equiv G' m_p m_e / \hbar c = 5.5982 \times 10^{-42} \equiv 2\gamma^* \tag{19}$$

$$\gamma_0 \equiv G_0 m_p m_e / \hbar c = 3.2155 \times 10^{-42} \equiv (1/1.7410)\gamma' \tag{20}$$

$$\gamma^* \equiv G^* m_p m_e / \hbar c = 2.7991 \times 10^{-42} \equiv (1/2)\gamma' \tag{21}$$

Accordingly, we can write the exact form of equations (6)–(6c) as

$$2^{1/\alpha} = \frac{1}{\gamma'} = \frac{\gamma'}{\alpha} N = 1.7863 \times 10^{41} \tag{6'}$$

which implies

$$\frac{R^*}{R_e} = \frac{1}{\gamma'} \tag{6a'}$$

$$N = \alpha (2^{1/\alpha})^2 = 2.3285 \times 10^{80} \tag{6b'}$$

$$\alpha = N \gamma'^2 \tag{6c'}$$

In addition, these prime equations imply the exact relation

$$\frac{R_0}{R_e} = \frac{1}{\gamma_0} - \frac{1}{\gamma'} \tag{6d}$$

Moreover, since the electron Compton wavelength, $\lambda\!\!\!^{-}_e \equiv \hbar / m_e c$, is the fundamental dimensional-analytic length, it is rather nice that equations (6a) and (6d) imply

$$\frac{R^*}{\lambda\!\!\!^{-}_e} = \frac{\alpha}{\gamma'} \quad \text{and} \quad \frac{R_0}{\lambda\!\!\!^{-}_e} = \frac{\alpha}{\gamma_0} - \frac{\alpha}{\gamma'} \tag{6e}$$

Thus we are postulating a very restrictive (and interlocking) set of exact relations, which we can check immediately by calculating the universal present-day density, ρ_0, and comparing it with the $\rho_0 \simeq 7.1 \times 10^{-31}$ g cm^{-3} measured by Mathews and Viola (1979).

The calculation of ρ_0 is straightforward. We have derived (6′) by defining R^* as $2GM/c^2$. Since our changing-G formalism of equations (11)–(16) requires that the G of this definition be G^*, we can write

$$R^* \equiv 2G^* M / c^2 = 2G^* m_p N / c^2 = 5.0337 \times 10^{28} \text{ cm} \tag{22}$$

where $N = 2.3285 \times 10^{80}$ as in equation (6b′). Therefore, using equation (18)

$$R_0 = (0.7410)R^* = 3.730 \times 10^{28} \text{ cm} \tag{23}$$

As a consistency check, using equation (6d), we note that

$$R_0 = \frac{e^2}{m_e c^2}\left(\frac{\hbar c}{G_0 m_p m_e} - 2^{1/\alpha}\right) = 3.730 \times 10^{28} \text{ cm} \tag{23′}$$

Thus we calculate the present-day universal density as

$$\rho_0 \equiv m_p N/\pi^2 R_0^3 = 7.604 \times 10^{-31} \text{ g cm}^{-3} \tag{24}$$

Note that the volume formula in equation (24) is for elliptical space, which identifies antipodal points and thus has half the volume of spherical space. In order to provide a basis for this as well as our definition of $R_{\max}$ we must develop a new formalism.

3. THE NEO-FRIEDMANN FORMALISM

The calculated density of 7.604×10^{-31} g cm^{-3}, so strikingly in accord with the Mathews and Viola (1979) "best guess" measurement of 7.1×10^{-31} g cm^{-3}, is critically dependent on the use of the gravitational radius as $R_{\max} \equiv R^* = 2M^*$, where $M^* \equiv G^*M/c^2$. Since the Friedmann formalism defines $R_{\max}$ as $R_f = (4/3)\pi M^*$, where M^* is $G_0 M/c^2$, our calculation of ρ_0, which is derived by postulating a closed universe, suggests that the primary cause of the "missing mass" problem is the use of a maximum radius, R_f, which is too small.

An analysis of the Friedmann formalism reveals that the definition of $R_{\max}$ as $(4/3)\pi M^*$ ultimately derives from the constant of proportionality, 8π, in Einstein's field equation,

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{25}$$

where ten functions of radius r in $G_{\mu\nu}$ are equated with ten functions of density ρ in $T_{\mu\nu}$. The presence of the 8π scalar implies that the ρ's are contained in volume elements of $(4/3)\pi r^3$. This is in keeping with Einstein's metric axiom which requires the elements of space to be Euclidian and thus in correspondence with Newtonian theory (Misner, Thorne, and Wheeler, 1973).

The Friedmann formalism is ordinarily derived by using Einstein's original form of equation (25) in which there is no cosmological term, $\Lambda g_{\mu\nu}$. Then by assuming that the universe in large scale is isotropic and homogenous, and that pressure can be neglected, the ten equations implicit in equation (25) can be reduced to one equation:

$$G_{tt} = 8\pi T_{tt} \tag{26}$$

where $T_{tt} \equiv \rho$ is interpreted as the density of the universe as a whole, rather than the density of a volume element. However, whereas the volume element can be considered to be Euclidian, the volume of the universe, in general, can not. Therefore, the standard Friedmann formalism would seem to be in error, since it incorporates the 8π term directly into the Friedmann equation of motion.

This equation of motion is simply equation (26) written out explicitly and algebraically rearranged:

$$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{8\pi}{3}\left(\frac{\rho R^2 G}{c^2}\right) - k \tag{27}$$

so that for the closed Friedmann cosmology (where $k = 1$), the definition of density which employs the spherical volume formula, $\rho = M/2\pi^2 R^3$, leads directly to the definition of $R_{\max} = (4/3)\pi M^*$ (where M^* is GM/c^2), if we write equation (27) in the cycloidal form:

$$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{R_{\max}}{R} - 1 \tag{28}$$

Thus it seems reasonable that, since there is observational conflict with the closed Friedmann cosmology, we should attempt to rectify the inconsistency of incorporating the 8π term (which is a Euclidian volume scalar) into the context of a non-Euclidian cosmological volume.

A consistent procedure would incorporate the necessary step, which the Friedmann formalism has omitted in going from the field equation (26) to the cosmological equation of motion (27). This necessary step must put the scalar term in a form that goes from the Euclidian volume element of equation (26) to the appropriate cosmological volume form.

Such a neo-Friedmann scalar should be written differently for positive, negative and flat universal curvature. This scalar is straightforwardly derivable in three forms from the appropriate volume formulas; and it is to be noted that in the case of positive curvature, it is necessary to employ the formula for elliptical space (where χ is integrated over 0 to $\pi/2$ rather than over 0 to π):

$$V=\int_0^{\pi/2} 4\pi R^2 \sin^2\chi (R\,d\chi)=\pi^2 R^3 \tag{29}$$

where χ is the covariant comoving coordinate related to the radius of curvature, R, by

$$r=R\sin\chi \tag{30}$$

$$D=R\chi \tag{31}$$

and where r is the radial coordinate defined by area $=4\pi r^2$, and D is the measured radial distance (or proper distance).

From equations (29) and (30) we can construct a volume function of χ which goes from π^2R^3 to $(4/3)\pi r^3$, as $r/R=\sin\chi$ goes from 1 to 0:

$$V(\chi)=\frac{(2\chi-\sin 2\chi)}{\sin^3\chi}\pi r^3 \equiv \varepsilon_+(\chi)\pi r^3 \tag{32}$$

where the positive neo-Friedmann function, $\varepsilon_+(\chi)$, goes from π to 4/3. (See Figure 1.)

Analysis of (32) shows that for fractions of the universe smaller than $(0.8)R$, the Euclidian volume is a good approximation, which improves as r/R approaches 0.

We note in passing that, in addition to cogent geometrical arguments for a cosmological elliptical—as opposed to spherical—volume (Weyl, 1952; Eddington, 1924), we must consider that $\varepsilon_+(\chi)$ of equation (32) depends on the use of the elliptical volume equation (29). Thus the use of this volume in the density calculation of equation (24) is justified by consistency.

From the definition of hyperbolic volume, we can similarly derive the negative neo-Friedmann function, $\varepsilon_-(\chi)$. Thus we can write the family of neo-Friedmann function:

$$\text{for } k=+1, \quad \varepsilon_+(\chi)=\frac{2\chi-\sin 2\chi}{\sin^3\chi} \tag{33a}$$

$$\varepsilon\ (\chi\ )=\text{for } k=0, \quad \varepsilon_0(\chi)=\frac{4}{3} \tag{33b}$$

$$\text{for } k=-1, \quad \varepsilon_-(\chi)=\frac{\sinh\chi-2\chi}{\sinh^3\chi} \tag{33c}$$

where $0<\varepsilon_-(\chi)<4/3$; and $4/3<\varepsilon_+(\chi)\leqslant\pi$.

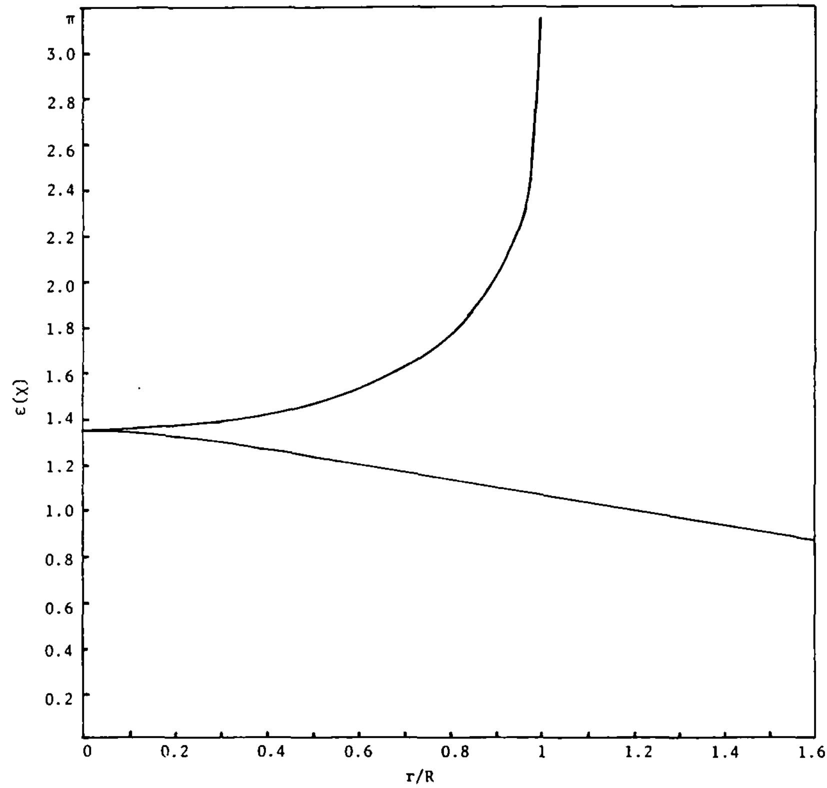

Fig. 1. The neo-Friedmann function: $0 < \varepsilon(\chi) < \pi$ for $0 < \chi < \infty$. The upper curve: $k = +1$, $r/R = \sin\chi$, $\varepsilon_{+}(\chi) = (2\chi - \sin 2\chi)/\sin^3\chi$. The lower curve: $k = -1$, $r/R = \sinh\chi$, $\varepsilon_{-}(\chi) = (\sinh 2\chi - 2\chi)/\sinh^3\chi$. The zero point: $k = 0$, $r/R = 0$, $\varepsilon_0(\chi) = 4/3$.

The neo-Friedmann cosmology is then defined as that set of equations which is consistent with the cosmologically interpreted equation:

$$G_{tt} = \varepsilon(\chi) 6\pi T_{tt} \tag{34}$$

Since we are postulating a closed universe, we need to consider here only the $\varepsilon_{+}(\chi)$ form of equation (34). From the definition of $R_{\max} = 2M^*$, implicit in equations (32)–(34), and from the model of G changing as an inverse function of R, worked out in equations (11)–(16), it is possible to construct a self-consistent formalism. Because of its similarity to the closed Friedmann cosmology, I call this formalism the closed neo-Friedmann

TABLE I. The Neo-Friedmann Formalism Compared with the Friedmann Formalism

Closed Friedmann cosmology ($\Lambda = 0$; $k = 1$; $R_{max} = (4/3\pi)M^*$)[a]	Closed neo-Friedmann cosmology ($\Lambda = 0$; $k = 1$; $R_{max} = 2M^*$)	
$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{8}{3}\pi\rho R^2 G/c^2 - 1$	$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \pi^2\rho R^2 G'/c^2 - 1$	(35)
$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{R_f}{R} - 1$	$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 = \frac{R^*}{R} - 1$	(36)
$R_f \equiv R_{max} = \frac{4MG}{3\pi c^2} \equiv \frac{4}{3\pi}M^*$	$R^* \equiv R_{max} = \frac{2MG^*}{c^2} \equiv \frac{MG'}{c^2} \equiv 2M^*$	(37)
$G = 6.6720 \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}}$	$G_0 = 6.6720 \times 10^{-8}\ \mathrm{cm^3\ g^{-1}\ s^{-2}}$	(38)
	$G' = f_0 G_0 = fG = 2G^*$	(39)
	$f = \frac{R}{R^*} - 1$	(40)
$R = \frac{R_f}{2}(1 - \cos\eta)$	$R = \frac{R^*}{2}(1 - \cos\eta)$	(41)
$t = \frac{R_f}{2c}(\eta - \sin\eta)$	$t = \frac{R^*}{2c}(\eta - \sin\eta)$	(42)
$H = \cot\left(\frac{\eta}{2}\right)\frac{c}{R}$	same	(43)
$M = 2\pi^2 R^3 \rho$	$M = \pi^2 R^3 \rho$	(44)
$R = c\left(\frac{8}{3}\pi G\rho - H^2\right)^{-1/2}$	$R = c(\pi^2 G'\rho - H^2)^{-1/2}$	(45)
$\Omega \equiv \frac{\rho}{\rho_{crit}} = \frac{8\pi G\rho}{3H^2}$	$\Omega \equiv \frac{\rho}{\rho_{crit}} = \frac{\pi^2 G'\rho}{H^2}$	(46)
$q \equiv \frac{1}{2}\Omega \equiv (1 + \cos\eta)^{-1}$	$q \equiv \frac{1}{2}\Omega \equiv (1 + \cos\eta)^{-1} = \frac{\pi^2 G^*\rho}{H^2}$	(47)

[a] Note: Λ is the cosmological constant, k is the curvature constant, and $M^* \equiv GM/c^2$.

cosmology. Table I describes explicitly the similarities and differences between these two closed cosmologies.

4. THE CHANGING G

The neo-Friedmann formalism raises the question: is equation (16), in which G varies as an inverse function of R, consistent with the cycloid parameter equations (41)–(43)? Since these equations are implicit in the equation of motion (36), we are immediately struck by the similarity in form between equations (16) and (36). Since, by using $G' = 2G^*$, equation (16) is

$$\frac{R^*}{R} = \frac{2G^*R^*}{GR} - 1 \tag{16'}$$

likewise we can rewrite (36) as

$$\frac{1}{2c^2}\left(\frac{dR}{dt}\right)^2 = \frac{G^*R^*}{GR} - 1 \tag{36'}$$

Thus the form of the changing-G equation (16′) fits perfectly the form of the neo-Friedmann equation of motion (36′). And therefore (16′) also fits the cycloid parameters, which are the solutions of the equation of motion (36).

It should especially be noted that the basic form of the neo-Friedmann equation of motion (35) employs G' (which does not change). G' is also used in the equation for R, (45), and in the density parameter equation (46). It is precisely this feature which makes possible a changing-G in the context of a formalism of the Friedmann type.

Canuto (1979) has analyzed the argument that a formalism with a changing G would be inconsistent with general relativity. The crux of this argument is that general relativity requires the product GM to be constant, and that if conservation of mass–energy is maintained, G must be constant. In the neo-Friedmann formalism, however, we have seen that the constant G', operative in the key equations, does not change, so that $G'M$ does not vary.

Moreover, as Misner, Thorne, and Wheeler (1973) point out, the equation of motion for the closed cosmology, especially in the cycloidal form

$$\frac{1}{c^2}\left(\frac{dR}{dt}\right)^2 - \frac{R_{\max}}{R} = -1 \tag{28'}$$

can be considered as an analog to the classical statement of conservation of energy. The first term is like kinetic energy, and the second term is like potential energy.

The neo-Friedmann form of equation (28) employs the changing G, but in such a way that the constancy of the equation is maintained:

$$\frac{1}{2c^2}\left(\frac{dR}{dt}\right)^2 - \frac{G^*R^*}{GR} = -1 \tag{36''}$$

Thus we find that this very special form of G, which changes as an inverse function of R, is consistent with general relativity, especially as this theory is applied to the universe as a whole.

The definitive test of this changing-G formalism will, of course, be the detection of a change in G of the magnitude predicted by the closed neo-Friedmann cosmology. From equations (16), (41) and (42) it is possible to construct an explicit model of the rate of change of G, as in Table II.

It must be noted that the neo-Friedmann calculation of the current annual rate, $\dot{G}/G_0 = -2.2\times10^{-12}\ \mathrm{yr}^{-1}$, differs by more than an order of magnitude from the Dirac prediction of $-6\times10^{-11}\ \mathrm{yr}^{-1}$. Wesson (1980) reviews experimental evidence which seems to confirm the Dirac rate. However, McElhinny, Taylor, and Stevenson (1978), using paleomagnetic tests of earth's expansion during the past 400 Myr, cite an upper limit of $-8\times10^{-12}\ \mathrm{yr}^{-1}$, which would rule out Dirac's proposal, but remain consistent with the neo-Friedmann value of $-6.3\times10^{-12}\ \mathrm{yr}^{-1}$ during the past 500 Myr, as in Table II. Clearly, our methods of measuring $\dot{G}/G_0$ will require significant improvement before we are in a position to confidently

TABLE II. Predicted Rate of Change of G over Various Epochs

Δt ($\times10^9$ yr)	G ($\times10^{-8}\ \mathrm{cm^3\,g^{-1}\,s^{-2}}$)	ΔG ($\times10^{-8}\ \mathrm{cm^3\,g^{-1}\,s^{-2}}$)	$\frac{\Delta G}{\Delta t}\cdot\frac{1}{G_0}\to\frac{\dot{G}}{G_0}$
31.872[a]	11.616	−4.9440	$-23.2\times10^{-12}\ \mathrm{yr}^{-1}$
10.000	7.2059	−0.5339	$-8.0\times10^{-12}\ \mathrm{yr}^{-1}$
5.000	6.9087	−0.2367	$-7.1\times10^{-12}\ \mathrm{yr}^{-1}$
1.000	6.7150	−0.0430	$-6.4\times10^{-12}\ \mathrm{yr}^{-1}$
0.500	6.6930	−0.0210	$-6.3\times10^{-12}\ \mathrm{yr}^{-1}$
0.250	6.6822	−0.0102	$-6.1\times10^{-12}\ \mathrm{yr}^{-1}$
0.100	6.6757[b]	−0.0038	$-5.6\times10^{-12}\ \mathrm{yr}^{-1}$

(This calculated series implies the limiting value $\dot{G}/G_0 = -2.2\times10^{-12}\ \mathrm{yr}^{-1}$.)

[a] The age of the universe in this neo-Friedmann cosmology.
[b] Note that G today, G_0, is $6.6720\times10^{-8}\ \mathrm{cm^3\,g^{-1}\,s^{-2}}$.

confirm or deny the neo-Friedmann prediction of a current rate of -2.2×10^{-12} yr^{-1}.

5. THE HUBBLE PARAMETER, H_0

Measurement of the current Hubble parameter is a less problematic test of the neo-Friedmann cosmology. Combining equations (22) and (23) with equation (41) implies a cycloid parameter, $\eta_0 = 2.074$ radians, and thus using equation (43) we can calculate $H_0 = 4.753\times 10^{-19}$ s^{-1}, which is equivalent to 14.66 km s^{-1} Mpc^{-1}. Since this value is less than any of its measurements to date, precise measurement of H_0 will provide a stringent test of the neo-Friedmann model presented here. The difficulty in measuring H_0 lies largely in its dependence on long chains of distance parameters (Weinberg, 1972). With the refinement of distance scales since the initial (Hubble, 1929) determination of this parameter as 513 km s^{-1} Mpc^{-1}, H_0 has systematically been revised downward by more than an order of magnitude.

A widely cited recent measurement of H_0 is the Sandage and Tammann (1976) value of 50.3 km s^{-1} Mpc^{-1}. I can demonstrate, however, that their paper contains a hint of a drastic future lowering of H_0. Sandage (1975) and others have determined H_0 by using giant E galaxies as "standard candles" in deriving the red-shift : distance relation. The key unknown in this method is the absolute magnitude of the giant E galaxies, which depends on the distance to the Virgo cluster, since that is the site of the nearest giant E galaxy. There are several ways to determine the Virgo cluster distance scale, but the most straightforward uses Type I supernovae in the Virgo cluster as standard candles. And it is precisely this method which provides the conspicuously large Virgo cluster distance modulus of 32.91 as reported by Sandage and Tammann (1976). Their H_0 of 50.3 km s^{-1} Mpc^{-1} is calculated by averaging this enormous distance modulus with several lower values, derived by more indirect methods. Moreover, they use E galaxy red-shift : magnitude data that was summarized in Sandage (1972), but had already been superceded by the more precise data of Kristian, Sandage, and Westphal (1978). Curiously, this high-quality data—derived from using both SIT photometry and SIT spectroscopy—when combined with the Virgo distance modulus of 32.91, yields an H_0 of 16.5 ± 1.6 km s^{-1} Mpc^{-1}.

There are reports (Vaucouleurs and Bollinger, 1979; Aaronson et al., 1980) of H_0 at double the Sandage–Tammann rate, which serves mainly to emphasize the inconclusiveness of our current measurements. However, as the quality of the relevant data improves, especially when the NASA Space Telescope (Tammann, 1979) becomes operational in the mid-1980s, we should not be too surprised to see H_0 approach the neo-Friedmann value of 14.66 km s^{-1} Mpc^{-1}.

6. THE DECELERATION PARAMETER, q_0

Sandage (1970) has characterized cosmology as "a search for two numbers"; and for him these numbers are H_0, as described above, and q_0, the deceleration parameter, or the rate at which the expansion is changing today. By definition

$$q_0 \equiv \frac{\ddot{R}_0 R_0}{\dot{R}_0} \tag{48}$$

Of course, q_0 is much more difficult to measure than H_0, because observations at greater distances are entailed in its determination. But q_0 is the most relevant variable in deciding whether the universe is open or closed in the sense of whether it will go on expanding forever or not. The critical value is $1/2$. If $q_0 > 1/2$, the universe will attain a maxim radius of curvature and then collapse. In Friedmann cosmologies, this also implies that the universe is finite and closed in the sense of being topologically compact.

In a closed Friedmann-type cosmology q_0 is easily calculated from

$$q_0 = (1 + \cos \eta_0)^{-1} \tag{49}$$

so that since from equations (22), (23), and (41), η_0 is 2.074 radians, the neo-Friedmann calculation of q_0 is 1.93.

Support for this value can be found in the recent measurements of Baldwin, Burke, Gaskel, and Wampler (1978), who report $q_0 \simeq 2$. However, this work entails a new method of measuring what is by all accounts an extremely elusive quantity, and so this supporting value must be cited with due caution. Only further observations will tell.

It must be emphasized at this point that the literature on q measurement is sometimes misleading, because observational data (Weinberg, 1972; Sandage, 1972) is presented with alternative q lines bending up and down, in the log : log space of the Hubble diagram, and it is thus implied that we are waiting for further data to see how the $z : d_L$ (or $z : m$) line bends in order to determine q. As a matter of fact, however, if we assume a Friedmann universe—whether open or closed—all the q lines in the Hubble diagram are straight lines of different slope. This is illustrated by explicit calculation in Figure 2.

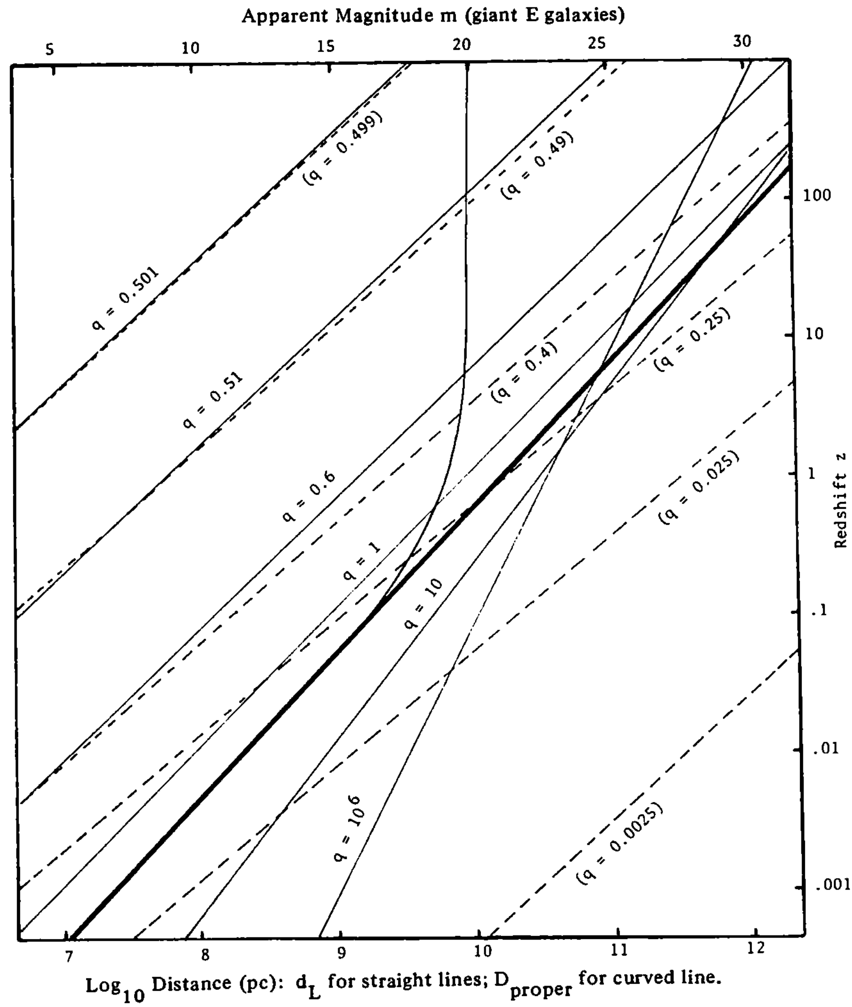

Fig. 2. Red-shift: distance graphs for Friedmann-type ($\Lambda = 0$) cosmology, with R^* fixed at 5.0337×10^{28} cm. Solid q lines are for different epochs of a closed cosmology. Dotted q lines are for an open cosmology. Heavy line is for $q_0 = 1.93$.

The family of solid-line q graphs in Figure 2 represents a closed cosmology. Each of these q lines is determined by selecting q and calculating the chain: $\eta = \cos^{-1}(1/q - 1)$, $R = (1 - \cos\eta)R^*/2$, $1/H = \tan(\eta/2)(R/c)$. Then select a set of η_1 for which $R_1 = (1 - \cos\eta_1)R^*/2$, $z = (R - R_1)/R_1$, $d_L = (c/Hq^2)\{zq + (q-1)[(2zq+1)^{1/2} - 1]\}$. It is to be noted that each such q graph is a straight line representing an epoch, $t(\eta)$, during which the shape of the Hubble diagram would remain as in Figure 2, but with the d_L and m values shifted to the right or to the left.

The Friedmann and neo-Friedmann cosmologies do differ in their predictions of density, given an identical $R_{\max}$. For example, the closed neo-Friedmann density prediction of this paper is 7.61×10^{-31} g cm^{-3}, whereas, for the same $R_{\max}$, the closed Friedmann density prediction is 1.56×10^{-30} g cm^{-3}, which is just outside the range of the Mathews and Viola (1979) measurement of ρ_0.

The predictive differences of these two cosmologies may not seem great, yet the conceptual difference is very great, as illustrated by the ability of the neo-Friedmann formalism to provide a way of determining $R_{\max} \equiv R^*$. Other conceptual differences are explicitly detailed in Table I, which formalizes the G-changing aspect of the neo-Friedmann cosmology. The specific predictions of this changing-G are contained in Table II, which predicts a current annual rate of -2.2×10^{-12} yr^{-1}. The other predictions of the closed neo-Friedmann cosmology are summarized in Table III.

7. COMPARISON WITH EDDINGTON

Since Eddington (1935, 1936, 1946) was one of the earliest and most prolific writers on the implications of the large-number coincidences in physics, this paper can hardly be complete without comparing our approach with his.

Essentially, the approach of this paper is the inverse of Eddington's. For he builds up a model of the universe from which he derives the physical constants; whereas, we build up a model of the universe as a function of the physical constants. In both cases the large-number coincidences are used as clues for model building, but quite different models are constructed.

Our belief is that Eddington's goal of accounting for the values of the physical constants is achievable only after a model such as ours is worked out and confirmed observationally. That is, we must be sure we know what role the physical constants play before we can account for the particular values they have.

An analysis of our model (the neo-Friedmann formalism) shows that the values of c, $\hbar$, e, and m_e are sufficient to determine R^*. Therefore, as we have shown in Figure 2, the complete family of q lines for the universe is determined by just these constants. In order to select a particular q line, we must know (in addition to these constants) the values of G and m_p. Since G is the only variable in this model, we would like to have a fundamental formula for the mass ratio m_p/m_e. Eddington (1936) has proposed such a formula, but it is generally considered to be flawed. This author (1977) has suggested the alternative, more straightforward combinatorial formula: $m_p/m_e \simeq P_2^{136}/10 = 1836$. If we take this formula into account, we can say that in the model presented here, it is essentially G which selects the epoch.

Future generalizations of this model could, of course, make other parameters, such as m_e (or combinations, such as α) into variables. In this way, it might be possible to begin the construction of a fundamental account of the physical constants, such as Eddington attempted.

8. DISCUSSION

It has sometimes been said that the first law of cosmology is: $1 = 10$. This "law" is invoked implicitly in most discussions of the "large-number hypothesis." We have found in the present paper, however, that a better strategy is to follow up the realization that on a large scale things are most likely to be symmetric and simple, and that therefore on the cosmic scale one can hope for a measure of precision unobtainable elsewhere. Just as Eratosthenes (c. 250 B.C.) could successfully calculate the radius of curvature of the surface of the earth by making symmetrizing assumptions and ignoring local irregularities, so we today can hope to similarly calculate the radius of curvature of the universe. As soon as one asks: how can the large-number coincidence relations be put into a form that is exact but still in agreement with general relativity?—the results of this paper follow.

Eratosthenes had to wait for Magellan's sailors (in 1522) to confirm his calculation, therefore it is fortunate for us that our calculation of $R_0 = 12.09$ Gpc implies an H_0 of 14.66 km s^{-1} Mpc^{-1}, which is likely to be decisively confirmed or denied within less than a decade.

Intrinsic to the formalism of this paper is the neo-Friedmann function which governs the change from a Euclidian to a non-Euclidian volume.

Our central observational claim is the straight q-line Hubble diagram for Friedmann-type cosmologies, which is derived simply by fixing R^*.

Our central theoretical claim is that the large-scale structure of the universe can be derived from measurement of the fundamental physical constants: c, $\hbar$, e, and m_e setting the scale factor R^*, and G determining the epoch.

Only observation will tell whether all or any of this is true.

Postscript: Spinrad, Stauffer, and Butcher [*Astrophysical Journal*, **244**, 382–391 (1981)] have measured magnitudes at various frequencies for two extremely distant galaxies: 3C472.1 ($z = 1.175$); and 3C13 ($z = 1.050$). These apparent magnitudes are approximately 22 and 21, respectively. Thus these data fit the $q_0 = 1.93$ line of Figure 2, which also implies $H_0 = 14.66$

km s^{-1} Mpc^{-1}, and that the absolute magnitude of the standard candle giant E galaxies is around -25.

As stated above, this absolute magnitude will not be well established until the proper distance to the Virgo cluster (site of the closest such galaxy) is measured—in an H_0 independent fashion—by the Space Telescope around 1985.

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REFERENCES

Aaronson, M., Mould, J., Huchra, J., Sullivan III, W. T., Schommer, R. A., and Bothun, G. D. (1980). *Astrophysical Journal*, **239**, 12.

Baldwin, J. A., Burke, W. L., Gaskell, C. M., and Wampler, E. J. (1978). *Nature*, **273**, 431.

Bastin, T. (1971). In *Quantum Theory and Beyond*, Ted Bastin, ed., Cambridge University Press, London, p. 213.

Burger, T. J. (1978). *Nature*, **271**, 402.

Canuto, V. M. (1979). *New Scientist*, **81**, 856.

Cohen, E. R., and Taylor, B. N. (1973). *Journal of Physical Chemical Reference Data*, **2** (4), 663.

Dirac, P. A. M. (1937). *Nature*, **139**, 323.

Eddington, A. S. (1924). *The Mathematical Theory of Relativity, 2nd ed.*, Cambridge University Press, Cambridge, p. 157.

Eddington, A. S. (1935). *New Pathways in Science*, Cambridge University Press, Cambridge, p. 229.

Eddington, A. S. (1936). *Relativity Theory of Protons and Electrons*, Cambridge University Press, Cambridge, p. 212.

Eddington, A. S. (1946). *Fundamental Theory*, Cambridge University Press, Cambridge, p. 265.

Field, G. B. (1976). In *Frontiers of Astrophysics*, Eugene H. Avrett, ed., Harvard University Press, Cambridge, Massachusetts, p. 223.

Giacconi, R. (1980). *Scientific American*, **242** (2), 80.

Harrison, E. R. (1972). *Physics Today* **25** (12), 30.

Hawking, S. W., and Ellis, G. F. R. (1973). *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge, p. 117.

Hubble, E. (1929). *Proceedings of the National Academy of Science US*, **15**, 169.

Kristian, J., Sandage, A., and Westphal, J. A. (1978). *Astrophysical Journal*, **221**, 383.

Lang, S. (1970). *Algebra*, Addison-Wesley, Reading, Massachusetts, p. 367.

McElhinny, M. W., Taylor, S. R., and Stevenson, D. J. (1978). *Nature*, **271**, 316.

Mathews, G. J., and Viola Jr., V. E. (1979). *Astrophysical Journal*, **228**, 375.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). *Gravitation*, Freeman, San Francisco, pp. 417, 705, 851.

Reines, F., Sobel, H. W., and Pasierb, E. (1980). *Physical Review Letters*, **45**, 1307.

Salam, A. (1970). International Center for Theoretical Physics, Trieste, Preprint IC/70/1906.

Sandage, A. R. (1970). *Physics Today*, **23** (2), 34.

Sandage, A. (1972). *Astrophysical Journal*, **178**, 1.

Sandage, A. (1975). In *Galaxies and the Universe*, Vol. IX of *Stars and Stellar Systems*, Allan Sandage, Mary Sandage and Jerome Kristian, eds., University of Chicago Press, Chicago, p. 761.

Sandage, A., and Tammann, G. A. (1976). *Astrophysical Journal*, **210**, 7.

Sirag, S.-P. (1977). *Nature*, **268**, 254.

Tammann, G. A. (1979). In *Scientific Research with the Space Telescope, NASA CP-2111*, M. S. Longair and J. W. Warner, eds., U.S. Government Printing Office, Washington, D. C., p. 263.

Vaucouleurs, G., and Bollinger, G. (1979). *Astrophysical Journal*, **233**, 433.

Weinberg, S. (1972), *Gravitation and Cosmology*, Wiley, New York, p. 405.

Wesson, P. S. (1980). *Physics Today*, **33** (7), 32.

Weyl, H. (1949). *Philosophy of Mathematics and Natural Science*, Princeton University Press, Princeton, New Jersey, p. 289.

Weyl, H. (1952). *Space Time Matter*, Dover, New York, p. 273.